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## Does slow growth lead to rising inequality?

A stock-flow consistent exploration of the 'Piketty hypothesis'\*



**PASSAGE**  
Working Paper Series  
15-03

ISSN: 2056-6255

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\* This paper is a revised and updated version of the earlier PASSAGE working paper 14-02.



Prosperity and Sustainability in the Green Economy (PASSAGE) is a Professorial Fellowship held by Prof Tim Jackson at the University of Surrey and funded by the Economic and Social Research Council (Grant no: ES/J023329/1).

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## Publication

Jackson T and P Victor 2015. Does slow growth lead to rising inequality? - A stock-flow consistent exploration of the 'Piketty hypothesis'. *PASSAGE Working Paper 15/03*. Guildford: University of Surrey. Online at: [www.prosperitas.org.uk/publications.html](http://www.prosperitas.org.uk/publications.html)

## Acknowledgements

The authors gratefully acknowledge support from the Economic and Social Research Council (ESRC grant no: ES/J023329/1) for Prof Jackson's fellowship on prosperity and sustainability in the green economy ([www.prosperitas.org.uk](http://www.prosperitas.org.uk)) which has made this paper possible and for support from the Ivey Foundation for Prof Victor. The paper has also benefitted from comments on the work by Karl Aiginger, Fanny Dellinger, Ben Drake, Armon Rezai, two anonymous referees and several participants in a small working group of the WWWforEurope project.

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**Abstract**

This paper explores the hypothesis (most notably made by French economist Thomas Piketty) that slow growth rates lead to rising inequality. If true, this hypothesis would pose serious challenges to achieving 'prosperity without growth' or meeting the ambitions of those who call for an intentional slowing down of growth on ecological grounds. It would also create problems of social justice in the context of a 'secular stagnation'. The paper describes a closed, demand-driven, stock-flow consistent model of Savings, Inequality and Growth in a Macroeconomic framework (SIGMA) with exogenous growth and savings rates. SIGMA is used to examine the evolution of inequality in the context of declining economic growth. Contrary to the general hypothesis, we find that inequality does not necessarily increase as growth slows down. In fact, there are certain conditions under which inequality can be reduced significantly, or even eliminated entirely, as growth declines. The paper discusses the implications of this finding for questions of employment, government fiscal policy and the politics of de-growth.

**NB: This is revised and updated version of PASSAGE Working Paper No 14/02**

## Introduction

The French economist, Thomas Piketty (2014a), has received widespread acclaim for his book *Capital in the 21st Century*. Building on over 700 pages of painstaking statistical analysis, the central thesis of the book is nonetheless relatively straightforward to describe. Piketty argues that the increase in inequality witnessed in recent decades is a direct result of the slowing down of economic growth in modern capitalist economies. Under circumstances in which growth rates decline further, he suggests, this challenge would be exacerbated.

So, for example, any future movement towards a 'secular stagnation' (Gordon 2012, MGI 2015, OECD 2014) is likely to be associated with even greater inequality. Equally, any policies aimed at deliberately 'dethroning' the Gross Domestic Product (GDP) as an indicator of progress (Turner 2008) could have perverse impacts on the distribution of incomes. Likewise, any objective of 'degrowth' for ecological or social reasons (Latouche 2008, Kallis et al 2011, d'Alisa et al 2015) might be expected to have undesirable social outcomes.

Piketty's approach has not been without criticism. Some have taken issue with his theoretical approach (Taylor 2014, Barbosa-Filho 2014) whilst others have challenged some of his empirical assumptions, particularly regarding the parameter sigma ( $\sigma$ ) taken to represent the elasticity of substitution between labour and capital (Semieniuk 2014, Levine et al 2014). Nonetheless, it is clear that Piketty's hypothesis that a slowing down of growth increases structural inequality poses a particular challenge to those ecological economists who, from the earliest days of the discipline (Daly 1972, Meadows et al 1972), have been critical of society's 'GDP fetish' (Stiglitz et al 2009) and sought to establish alternative approaches (Daly 1996, Victor 2008, Jackson 2009, van den Bergh 2011, Rezai et al 2012) in which socio-economic goals are achieved without continual throughput growth. Certainly, the prospects for 'prosperity without growth' (Jackson 2009) would appear slim at best if Piketty's thesis were unconditionally true.

The aim of this paper is therefore to unravel the extent of this challenge in more detail. To this end, we develop a simple closed, demand-driven model of Savings, Investment and Growth in a Macroeconomic framework (SIGMA).<sup>1</sup> We then use SIGMA to test for the implications of a slowdown of growth on a) capital's share of income and b) the distribution of incomes in the economy. By adding a government sector to the model, we also able to explore the potential to mitigate regressive impacts through a progressive taxation system. The inclusion of a banking sector allows us to establish clear relationships between the real and the financial economy and discuss questions of household wealth. Our ultimate aim is to tease out the implications of our findings for the wider project of developing an 'ecological macroeconomics'. First, however, we outline the structure of Piketty's argument in more detail.

## Piketty's two 'fundamental laws' of capitalism

There are two core strands to Piketty's case. One of them (Piketty 2014a: 22-25) concerns the power that accrues increasingly to the owners of capital, once the distribution of both capital and income becomes skewed. The power of accumulated or inherited wealth to set the conditions for the rates of return to capital and labour increasingly favours the owners

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<sup>1</sup> A user-version of the SIGMA model is available online at <http://www.prosperitas.org.uk/sigma> to allow the interested reader to conduct their own scenarios.

of capital over wage-earners and reinforces the advantages of the rich over the poor. These arguments are of course relatively well-known from Marxist and post-Marxist critiques of capitalism (Buchanan 1982, Goodwin 1967, Giddens 1995).

Piketty's principal contribution, however, is to identify what he calls a 'fundamental force for divergence' of incomes, in the structure of modern capitalism (op cit: 25-27). In the simplest possible terms it relates to the relative size of the rate of return on capital  $r$  to the growth rate  $g$ . When the rate of return on capital  $r$  is consistently higher than the rate of growth  $g$ , it leads to an accumulation of capital by the owners of capital and this tends to reinforce inequality, through the mechanism described above.

Piketty advances his argument through the formulation of two 'fundamental laws' of capitalism. The first of these (Piketty 2014a: 52 et seq) relates the capital stock (more precisely the capital to income ratio  $\beta$ ) to the share of income  $\alpha$  flowing to the owners of capital. Specifically, the first fundamental law of capitalism says that:<sup>2</sup>

$$\alpha = r\beta, \tag{1}$$

where  $r$  is the rate of return on capital. Since  $\beta$  is defined as  $K/Y$  where  $K$  is capital and  $Y$  is income, it is easy to see that this 'law' is, as Piketty acknowledges, an *accounting identity*:

$$\alpha Y = rK. \tag{2}$$

Formally speaking, the income accruing to capital equals the total capital multiplied by the rate of return on that capital. Though this 'law' on its own does not force the economy in one direction or another, it provides the foundation from which to explore the evolution of historical relationships between capital, income and rates of return. In particular, it can be seen from this identity that for any given rate of return  $r$  the share of income accruing to the owners of capital rises as the capital to income ratio rises.<sup>3</sup>

It is the second of Piketty's 'fundamental laws of capitalism' (op cit: 168 et seq; see also Piketty 2010) that generates particular concern in the context of declining growth rates. This law states that in the long run, the capital to income ratio  $\beta$  tends towards the ratio of the savings rate  $s$  to the growth rate  $g$ , ie:

$$\beta \rightarrow \frac{s}{g} \text{ as } t \rightarrow \infty. \tag{3}$$

This asymptotic law suggests that, as growth rates fall towards zero, the capital to income ratio will tend to rise dramatically over the longer term – depending of course on what happens to savings rates. Taken together with the first law, this suggests that over the long term, capital's share of income is governed by the following relationship:

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<sup>2</sup> In what follows, we suppress specific reference to time-dependency of variables except where absolutely necessary. Thus all variables should be read as time dependent unless specifically denominated with a subscripted suffix 0. Occasionally, we will have reason to use the subscripted suffix (-1) to denote the first lag of a time-dependent variable.

<sup>3</sup> We will see later that the *ceteris paribus* clause relating to constant  $r$  here is important. In fact, the rate of return will typically change as the capital to income ratio rises; and to the extent that this ratio declines with increasing  $\beta$ , it can potentially mitigate the accumulation of the capital share of income.

$$\alpha \rightarrow r \frac{s}{g} \text{ as } t \rightarrow \infty. \quad (4)$$

In other words, as growth declines, the rising capital to income ratio  $\beta$  leads to an increasing share of income  $\alpha$  going to capital and a declining share of income going to labour. Unless the distribution of capital is itself entirely equal (a situation we discuss in more detail later) this relationship therefore presents the spectre of a rapidly escalating level of income inequality. Rising wealth inequality would also flow from this. Differential savings rates – in which higher income earners save proportionately more than lower income earners (or, equally, where there are lower propensities to consume from capital than from income) – would reinforce these inequalities further by allowing the owners of capital to accumulate even more capital and command even higher wages. The superior power of capital (op cit 22-25) then precipitates a rising structural inequality.

It is important to stress that the relationships (3) and (4) are long-term equilibria to which the economy evolves, provided that the savings rate  $s$  and the growth rate  $g$  stay constant. As Piketty points out, ‘the accumulation of wealth takes time: it will take several decades for the law  $\beta = s/g$  to become true’ (op cit: 168). In any real economy, the growth rate  $g$  and the savings rate  $s$  are likely to be changing continually, so that at any point in time, the economy is striving towards, but may never in fact achieve, the asymptotic result. Nonetheless, as Krusell and Smith (2014: 2) argue, equation (4) is ‘alarming because it suggests that, were the economy’s growth rate to decline towards zero, as Piketty argues it will, capital’s share of income could increase explosively’.

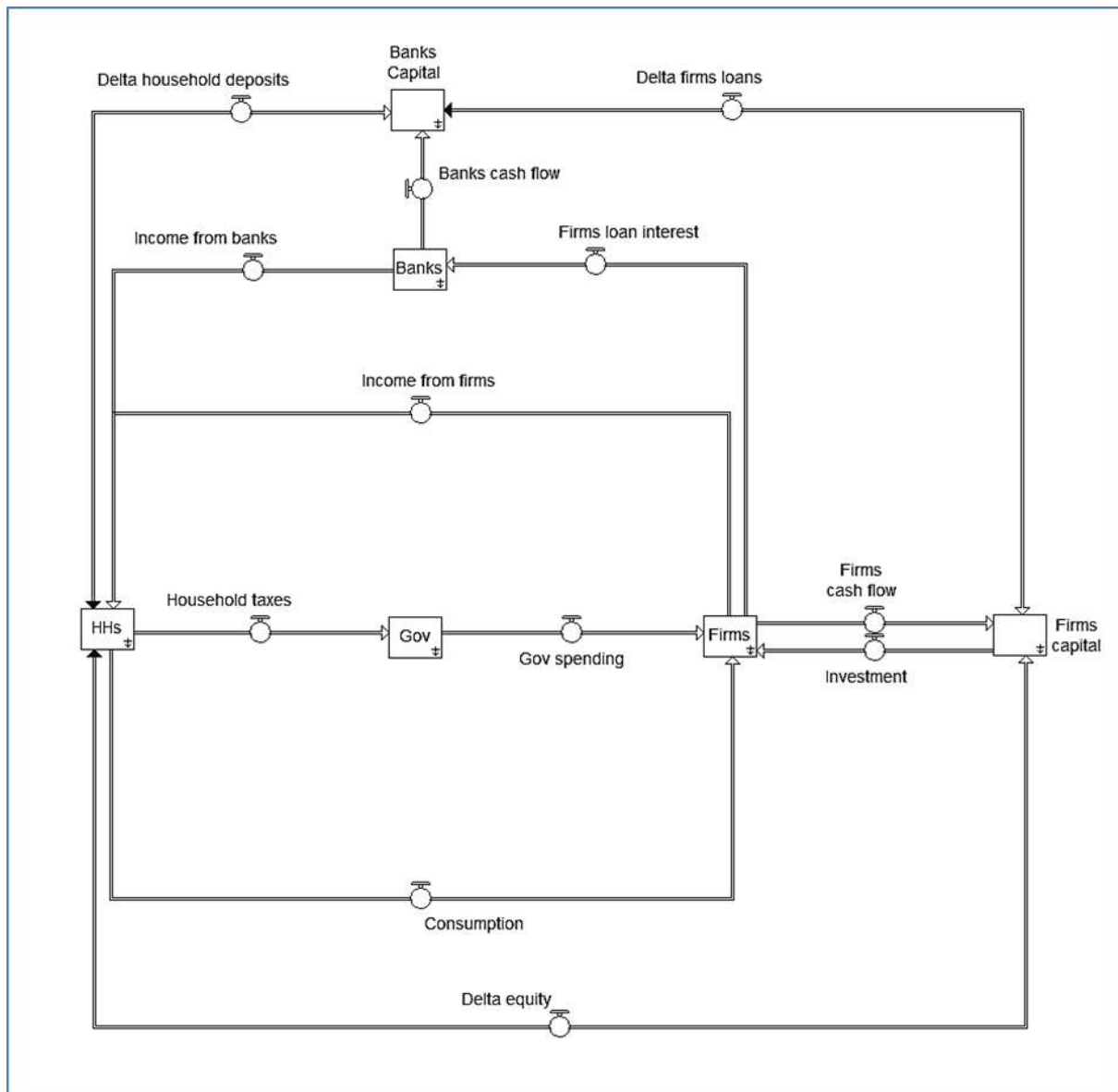
The principal aim of this paper is to test this hypothesis; ie to determine the extent to which declining rates of growth in national income,  $NI$ , and indeed in gross domestic product,  $GDP$ , might lead to rising capital to income ratios and thence to an increasing share of income to capital. In either formulation, much depends on the parallel movements in the rate of return on capital  $r$  and on the savings rate  $s$ . In order to explore these relationships in more detail, we built a simple, closed, stock-flow consistent (SFC), demand-driven model of savings, inequality and growth, calibrated loosely against UK and Canadian data. The background and structure for the model are described in the next section. The subsequent section presents our findings.

### **The SIGMA Model**

Working together over the last four years, the authors of this paper have developed an approach to macroeconomics which seeks to integrate ecological, real and financial variables in a single system dynamics framework (Jackson et al 2014, Jackson and Victor 2015.)

An important intellectual foundation for our work comes from the insights of post-Keynesian economics, and in particular from an approach known as Stock-Flow Consistent (SFC) macro-economics, pioneered by Copeland (1949) and developed extensively by Godley and Lavoie (2007) amongst others. The essence of SFC modelling is consistency in accounting for all monetary flows. Each sector’s expenditure is another sector’s income. Each sector’s financial asset is another’s liability. Changes in stocks of financial assets are consistently related to flows within and between economic sectors. These simple

understandings lead to a set of accounting principles which can be used to test the consistency of economic models. The approach has come to the fore in the wake of the financial crisis, precisely because of these consistent accounting principles and the transparency they bring to an understanding not just of conventional macroeconomic aggregates like the GDP but also of the underlying balance sheets. It has even been argued that the financial crisis arose, precisely because conventional economic models failed to take these principles into account (Bezemer 2011). Certainly, Godley (2007) was one of the few economists who predicted the crisis before it happened.



**Figure 1: High-Level Structure of the SIGMA model**

For the purposes of this paper, we have employed a simplified version of our overall approach. SIGMA is a closed, stock-flow consistent, demand-driven model of savings, inequality and growth in a macroeconomic framework. The model has four financial sectors: households, government, firms and banks (Figure 1). Firms’ and banks’ accounts are divided between current and capital accounts and the households sector is further subdivided into two subsectors (which we denominate as ‘workers’ and ‘capitalists’) in order to explore

potential inequalities in the distribution of incomes and of wealth. The model itself is built using the system dynamics software STELLA. This kind of software provides a useful platform for exploring economic systems for several reasons, not the least of which is the ease of undertaking collaborative, interactive work in a visual (iconographic) environment. Further advantages are the transparency with which one can model fully dynamic relationships and mirror the stock-flow consistency that underlies our approach to macroeconomic modelling.

Following much of the SFC literature, the model is broadly Keynesian in the sense that it is demand-driven. Our approach is to establish a level of overall demand through an exogenous growth rate,  $g$ , and to generate the level of investment through an exogenous savings rate,  $s$ . We then explore the impacts of changes in these variables over time on the income shares from capital and labour through an endogenous rate of return,  $r$ , on capital. To achieve this we employ a constant elasticity of substitution (CES) production function, not to drive output as in a conventional neoclassical model, but to derive the marginal productivity  $r_K$  of capital  $K$  and also to establish the labour employment associated with a given level of aggregate demand.<sup>4</sup>

To illustrate our arguments without unnecessary complications, we work with a simplified version of the more complex structure that we have developed elsewhere (Jackson and Victor 2015). First, as noted, the SIGMA economy is closed with respect to overseas trade. Next, we assume that government always balances the fiscal budget and holds no outstanding debt, so that government spending,  $G$ , is equal to taxes,  $T$ , levied only on households. Finally, we employ a rather simple balance sheet structure (Table 1), sufficient only to get a handle on changes in household wealth under different patterns of ownership of capital. Households assets are held either as deposits,  $D$ , in banks or as equities,  $E$ , in firms. The only other category of assets/liabilities are the loans,  $L$ , made by banks to non-financial firms. The banking sector plays a relatively straightforward role as a financial intermediary, providing deposit facilities for households and loans to firms. Clearly none of these assumptions is accurate as a full description of a modern capitalist economy, but all of them can be relaxed in more sophisticated versions of our framework and none of them obstructs our purposes in this paper.

	Households	Firms	Banks	Govt	Total
<b>Net financial assets</b>	<b>D+E</b>	<b>-L-E</b>	<b>L-D</b>	<b>-</b>	<b>0</b>
Financial Assets	D+E		L	-	D+E+L
Deposits	D			-	D
Loans	-		L	-	L
Equities	E			-	E
Financial Liabilities	-	L+E	D	-	L+E+D
Deposits	-		D	-	D
Loans	-	L		-	L
Equities	-	E		-	E

**Table 1: Balance Sheet for the SIGMA Economy**

<sup>4</sup> We are aware of course of the limitations of using a broadly neoclassical production function (Cohen and Harcourt 2003, Robinson 1953). However, retaining this aspect of Piketty's analysis allows us to compare our findings more directly with his.



In a closed economy (ie an economy with no foreign trade) the national income  $NI$  can be interpreted both as the total income in the economy:

$$NI = W + P + i \quad (5)$$

where  $W$  represents wages,  $P$  profits (including rents), and  $i$  net interest receipts, and also as the demand by households, firms and government for goods, services and (net) investment in fixed capital:

$$NI = C + G + I_{net} \quad (6)$$

where  $C$  is consumer spending,  $G$  is government spending and  $I_{net}$  is net investment. The gross domestic product is then given by:

$$GDP = NI_d + \delta_0 K = C + G + I, \quad (7)$$

where  $K$  is the value of the capital stock,  $\delta_0$  is a (fixed) depreciation rate and gross investment  $I$  is given by:

$$I = I_{net} + \delta_0 K. \quad (8)$$

Since the two methods of calculation in equations (5) and (6) both lead to an equivalent net national income, it follows that:

$$W + P + i = C + G + I_{net}. \quad (9)$$

Profits  $P$  are generated both by nonfinancial firms and by banks. Banks profits  $P_b$  are simply the difference between the interest,  $i_f = r_l L_{-1}$ , charged to firms on loans and the interest,  $i_h = r_d D_{-1}$ , paid to households on deposits. We assume that banks distribute all of these profits to households. Nonfinancial firms on the other hand retain an exogenously determined proportion  $r_f$  of their total profits. Retained profits  $P_{fr}$  are then equal to  $r_f P_f$  and the remainder,  $P_{fd} = P_f - P_{fr}$  are distributed to households. Equation (9) can therefore be rewritten as:

$$W + P_b + P_{fd} + P_{fr} + i_h - i_f = C + G + I_{net}. \quad (10)$$

Since  $P_b = i_f - i_h$ , we can also write equation (9) as:

$$W + P_{fd} + P_{fr} = C + G + I_{net}, \quad (11)$$

and it becomes clear that in the SIGMA model at least, bank profits do not contribute to the national income which consists only in wages and firms profits. Furthermore, if we define the household income,  $Y_h^j$ , for each household type  $j$  according to:

$$Y_h^j = W^j + P_b^j + P_{fd}^j + i_h^j, \quad (12)$$

with  $j \in \{w, c\}$ , where  $w$  represents workers and  $c$  represents capitalists, then, equation (10) can be rewritten as:

$$Y_h^w + Y_h^c + P_{fr} - i_f = C + G + I_{net} \quad (13)$$

Noting that we can substitute  $T = T^w + T^c$  for  $G$  and  $C^w + C^c$  for  $C$  on the right hand side of equation (13), and rearranging terms, we find that:

$$I_{net} = (Y_h^w - C^w - T^w) + (Y_h^c - C^c - T^c) + (P_{fr} - i_f) \quad (14)$$

The first two terms in parentheses on the right hand side are, respectively, the savings  $S_h^w$  of workers and the savings  $S_h^c$  of capitalists, and the third term represents the savings  $S_f$  of nonfinancial firms. Accordingly, we can rewrite equation (14) as:

$$I_{net} = S_h^w + S_h^c + S_f \equiv S \quad (15)$$

where  $S$  is the total saving across the economy. Equation (15) is a special form of the so-called 'fundamental accounting identity' (Dorman 2014: 86) for a closed economy with a balanced fiscal budget. In SIGMA, the overall evolution of savings is determined by an exogenous savings rate,  $s$ , with respect to the national income, so that net savings across the economy are given by:

$$S = sNI \quad (16)$$

For the purposes of the exploration in this paper, we assume that  $s$  takes a fixed value  $s_0$  throughout each scenario. Since we are interested in the impact that different savings rates might have on different types of households, however, we allow the savings rate,  $s_w$ , of workers to be varied exogenously in different scenarios, so that the savings of worker households are given by:

$$S_h^w = s_w(Y_h^w - T^w). \quad (17)$$

In order to ensure that overall savings satisfy (16), the savings of capitalists are then determined as a balancing item.

$$S_h^c = S - S_h^w - S_f. \quad (18)$$

Household savings are distributed between new bank deposits,  $\Delta D$ , and the purchase of equities,  $\Delta E$ , from firms. It is assumed for simplicity that the demand for new equities by households is equal to the supply of new equities by firms and that these in their turn are determined via a desired debt to equity ratio in firms.<sup>5</sup> The distribution of equity purchases between capitalist and worker households is deemed to be in the same proportion as the net savings of each sector. Changes in deposits are then calculated as a residual from net savings.

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<sup>5</sup> In contrast to our treatment elsewhere (Jackson and Victor 2015), this means that there is no speculative purchasing of equities that might lead to capital gains and losses.

In order to model the evolution of the SIGMA economy over time, we start by defining the evolution of the net national income  $NI$  according to an (exogenous) growth rate  $g$  such that:

$$NI = (1 + g) * NI_{(-1)} \quad (19)$$

where  $NI_{(-1)}$  is the value in the previous period (ie the first lag) of the variable  $NI$ . In some scenarios  $g$  will take a fixed value  $g_0$  throughout the period  $\tau$  of the scenario,<sup>6</sup> while in others  $g$  will decline uniformly from  $g_0$  to zero over time  $t$ .

Testing Piketty's hypothesis requires that we establish the rate of return to capital,  $r$ , which in turn allows us to determine the split between wages and firms profits in the net national income. Along with Piketty (2014a: 213-214), we assume (for now) that the return to capital is given by the marginal productivity of capital, which we denote by  $r_K$ . This assumption only works under market conditions in which there are no structural features which might lead either capital or labour to extort more than their 'fair' share of the output from production. In a sense, this assumption is a conservative one for us, to the extent that conclusions about inequality are stronger in imperfect market dynamics. Under conditions of duress, in which the owners of capital receive a rate of return  $r$  greater than the marginal productivity of capital  $r_K$ , our conclusions about any inequality which results from declining growth rates will be reinforced. Conversely, of course, we must beware of making too strong assumptions about the potential to mitigate inequality, in any situation in which the owners of capital have greater bargaining power than wage labour.

With these caveats in mind, the next step is to determine the marginal productivity of the capital stock. In SIGMA, we achieve this through the partial differentiation of a constant elasticity of substitution (CES) production function of the form first developed by Arrow et al (1961) in which output,  $Y$ , is given by:

$$Y(K, L, \sigma) = (aK^{\frac{(\sigma-1)}{\sigma}} + (1 - a)(AL)^{\frac{(\sigma-1)}{\sigma}})^{\frac{\sigma}{(\sigma-1)}}, \quad (20)$$

where  $\sigma$  is the elasticity of substitution between labour and capital,  $a$  (as described by Arrow et al (1961) is a 'distribution parameter' and  $A$  is the coefficient of technology-augmented labour, which we will assume changes over time according to the rate of growth of labour productivity in the economy.<sup>7</sup> With a little effort, it can be shown via partial differentiation of equation (20) with respect to  $K$  that the marginal productivity of capital  $r_K$  is given by:

$$r_K = \frac{\partial Y}{\partial K} = a\beta^{\frac{-1}{\sigma}} \quad (21)$$

<sup>6</sup> In this paper we take  $\tau = 100$ , ie the scenarios run over 100 years.

<sup>7</sup> It can be shown that, for the special case  $\sigma = 1$ , this CES function reduces to the familiar Cobb-Douglas production function  $Y = K^a(AL)^{1-a}$ . The introduction of an explicit elasticity variable allows for a more flexible exploration of the production relationship under a variety of different assumptions about the elasticity of substitution between labour and capital.

where  $\beta$  is the capital to income ratio.<sup>8</sup> This relationship can now be used to derive the return to capital  $r_K K$  through:

$$r_K K = a\beta^{\frac{-1}{\sigma}} K \quad (22)$$

Taking the net national income  $NI$  as  $Y$ , and using Piketty's first law of capitalism (equation 2) it follows that capital's share of income  $\alpha$  is given by:

$$\alpha = a\beta^{\frac{\sigma-1}{\sigma}}. \quad (23)$$

It may be observed from equation (23), as Piketty also points out (2014b: 37-39), that for  $\sigma > 1$ , (and assuming that the capital to income ratio is greater than one) capital's share of income is an increasing function of the capital to income ratio. As the capital to income ratio rises, capital's share of income increases. Conversely however, when  $0 < \sigma < 1$ , then capital's share of income is a decreasing function of the capital to income ratio. As the share of capital to income rises, capital's share of income decreases. At  $\sigma = 1$ , the decline in the rate of return to capital always exactly offsets the rise in the capital to income ratio, and the share of income to capital remains constant. We explore the implications of these properties of equation (23) in the following section.

Armed with equation (23), we are now able to derive the profits of firms as:

$$P_f = r_K K = \alpha NI, \quad (24)$$

and calculate the income of worker and capitalist households from equation (12). Taxes are determined by exogenous tax rates on household income (and in some scenarios on household wealth), savings are determined through equations (16) to (18) and consumption can then be derived as a residual:

$$C^j = Y_h^j - T^j - S^j. \quad (25)$$

Equations (10) through (25) now allow for a full stock-flow consistent specification of the SIGMA economy. Table 2 summarises the flows within and between sectors in a single 'transaction flows matrix' (Godley and Lavoie 2007: 39). It is to be noted that all row totals and column totals in Table 2 sum to zero, reflecting principles of stock-flow consistency that each sector's expenditure is another sector's income (row totals) and that the sum of incomes and expenditures (including savings) in each sector must ultimately balance. It is also pertinent to observe that one of these sector balances has been left unspecified in equations (10) to (25): namely, the equation that balances banks' capital accounts:

$$\Delta L = \Delta D. \quad (26)$$

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<sup>8</sup> Note that as  $\sigma \rightarrow 1$ , this relationship returns to the 'first law' of capitalism (equation 1) with  $a = \alpha$ . In other words, under an assumption of unit elasticity of substitution between capital and labour (as in the Cobb Douglas function, the constant  $a$  is given by the share of income to capital  $\alpha$ .

	Households		Firms		Banks		Gov	$\Sigma$
	Workers	Capitalists	Curr	Cap	Curr	Cap		
Consumption (C)	$-C^w$	$-C^c$	$C$					0
Gov spending (G)			$G$				$-G$	0
Investment (I)			$I$	$-I$				0
Wages (W)	$W^w$	$W^c$	$-W$					0
Profits (P)	$+P_{fd}^w + P_b^w$	$+P_{fd}^c + P_b^c$	$-P_f$	$+P_{fr}$	$-P_b$			0
Taxes (T)	$-T^w$	$-T^c$					$T$	0
Interest	$+r_a D_{-1}^w$	$+r_a D_{-1}^c$	$-r_l L_{-1}$		$+r_l L_{-1} - r_a D_{-1}$			0
Change in deposits (D)	$-\Delta D^w$	$-\Delta D^c$				$+\Delta D$		0
Change in loans (L)				$+\Delta L$		$-\Delta L$		0
Change in equities (E)	$-\Delta E^w$	$-\Delta E^c$		$+\Delta E$				0
$\Sigma$	0		0	0	0	0	0	0

**Table 2: Transaction Flows Matrix for the SIGMA Economy**

Although  $\Delta L$  was defined via firms financing requirements and  $\Delta D$  was defined as the residual from household savings, the balance equation (26) is not in itself imposed as a constraint on the model. Rather, it should emerge as a result of all the other transactions in the economy, provided that the model itself is indeed stock-flow consistent (cf Godley and Lavoie 2007: 67-8). Equation 26 is therefore a useful check on the validity of the model as a whole. Since loans are created in the model as a financing demand, and deposits are a residual from household incomes, once all other outgoings are accounted for, we could also regard equation 26 as an illustration of the post-Keynesian claim that ‘loans create deposits’, in contradistinction to the claim of conventional monetary economics that ‘deposits create loans’. Indeed, it is possible to test this claim further by reducing the new loan requirements of firms (for instance by increasing the retained profits ratio) and observing that the level of new deposits in the economy does indeed decline.

In order to reflect the levels of inequality in different scenarios, we introduce a simple index of income inequality  $q_Y$  defined by:

$$q_Y = \left( \frac{Y_{dh}^c}{Y_{dh}^w} - 1 \right) * 100 \quad (27)$$

where  $Y_{dh}^c$  and  $Y_{dh}^w$  represents the disposable incomes of capitalists and workers (respectively). Note that in contrast to a more conventional index of inequality such as the Gini coefficient or the Atkinson index (Stymne and Jackson 2000, Howarth and Kennedy 2015 in this volume) our inequality index is unbounded. This choice allows us to illustrate numerically and graphically the divergence (or convergence) of incomes as growth declines. The index takes a value of 0 when the incomes of capitalists and workers are identical, ie there is no inequality at all, and a value of 100 when the income of capitalists is 100% higher (say) than that of workers. It can of course be considerably higher than 100 and we shall see this in some of the scenarios described in the following section.

For the purposes of exploring Piketty’s hypothesis that declining growth rates lead to rising inequality, the model described in this section is now complete. However, we note here that the production function in equation 20 can also be used to derive the labour requirements in the SIGMA economy, according to:

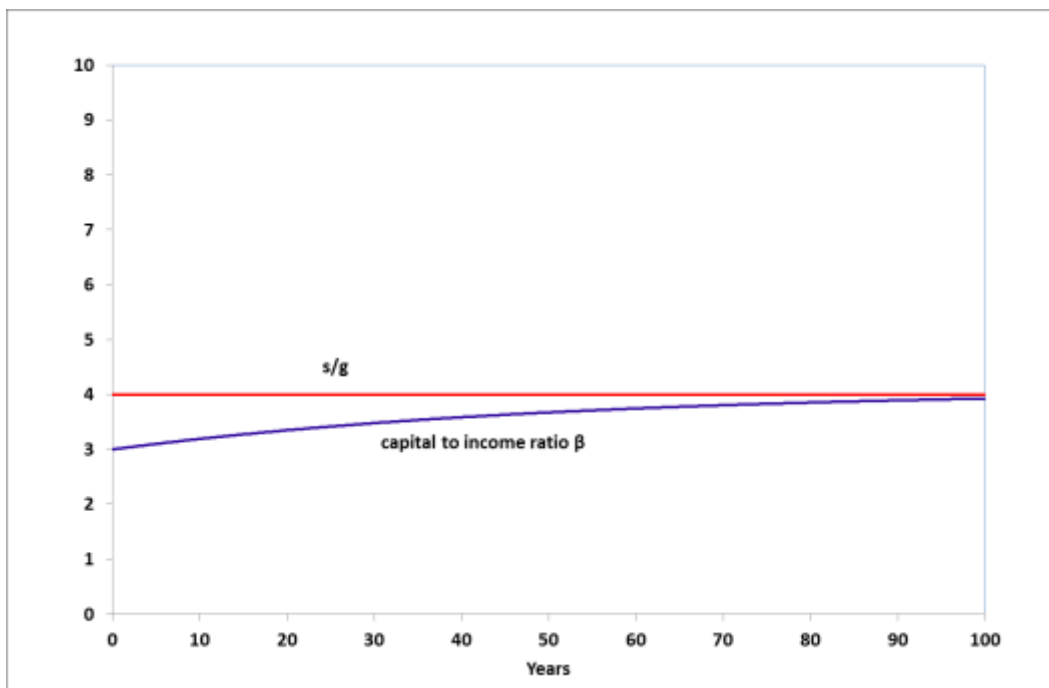
$$L = \frac{1}{A_t} \left( \frac{1}{1-a} \cdot \left( Y^{\frac{\sigma-1}{\sigma}} - aK^{\frac{\sigma-1}{\sigma}} \right) \right)^{\frac{\sigma}{\sigma-1}} \quad (28)$$

Since the pressure on unemployment is another of the threats from slower or zero growth, equation 28 will turn out to be a useful addition to the SIGMA model.

Our principal aim in this paper is conceptual. We aim to unravel the dynamics which threaten to lead to inequality under conditions of declining growth. SIGMA is therefore not inherently data-driven. Rather it aims to model the system dynamics that connect savings, growth, investment, returns to capital and inequality. It is nonetheless useful to ground the initial values of our variables in numbers which are reasonable or typical within modern capitalist economies (Table 3). Of particular importance, are reasonable choices for the initial values of the capital to income ratio, the savings rate and capital's share of income. For the purposes of this exercise we have therefore chosen representative values (Appendix 1) for the SIGMA variables, informed by empirical data for recent years.<sup>9</sup>

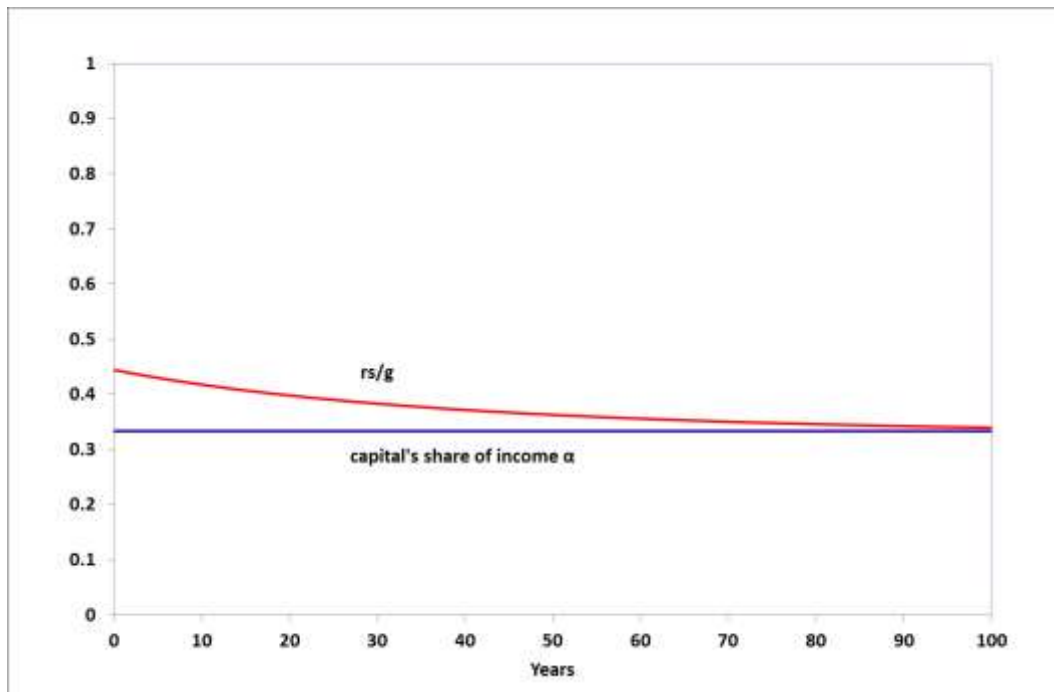
### Results

In the first instance, it is useful to illustrate the extent to which Piketty's 'laws of capitalism' hold true. Figure 2a shows the capital to income ratio ( $\beta$ ) and the ratio ( $s/g$ ) of savings rate to growth rate, when both  $s$  and  $g$  are held constant, for the values chosen in our reference scenario. Figure 2b shows capital's share of income ( $\alpha$ ) alongside the ratio  $rs/g$ , under the same conditions. For these conditions, it is clear both that the convergence predicted by Piketty occurs and also that this convergence takes some time (around a century in this case).



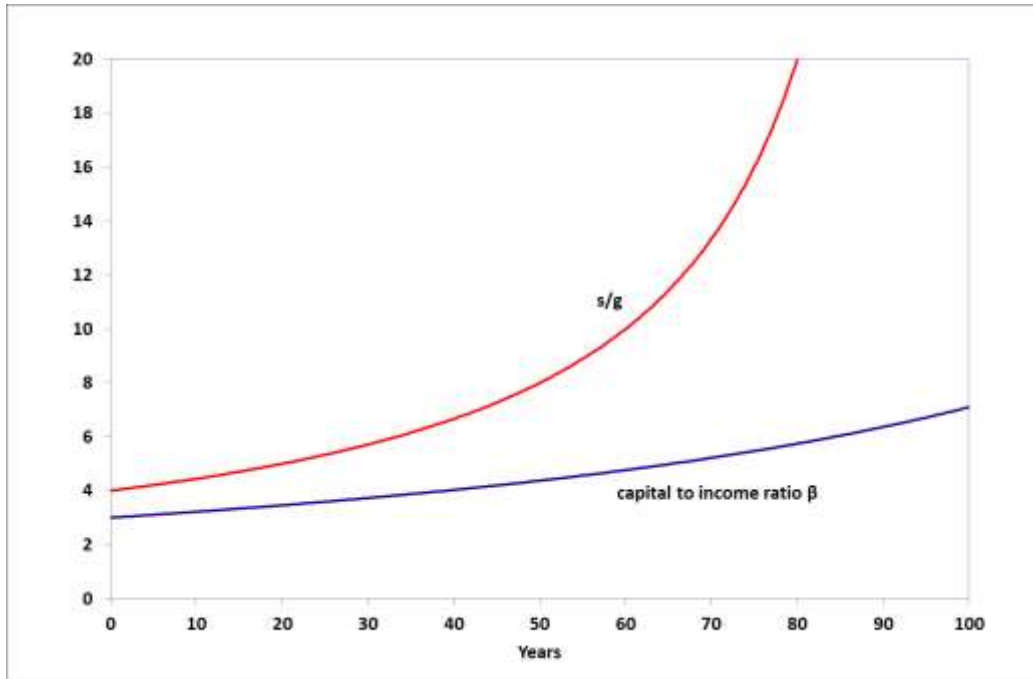
**Figure 2a: Long-term convergence of the capital to income ratio with  $s$  and  $g$  held constant**

<sup>9</sup> Data for the Canadian economy may be found in the Cansim online database: <http://www5.statcan.gc.ca/cansim/home-accueil?lang=eng>; and for the UK economy on the Office for National Statistics online database: <http://www.ons.gov.uk/ons/taxonomy/index.html?nsci=Economy#tab-data-tables>.

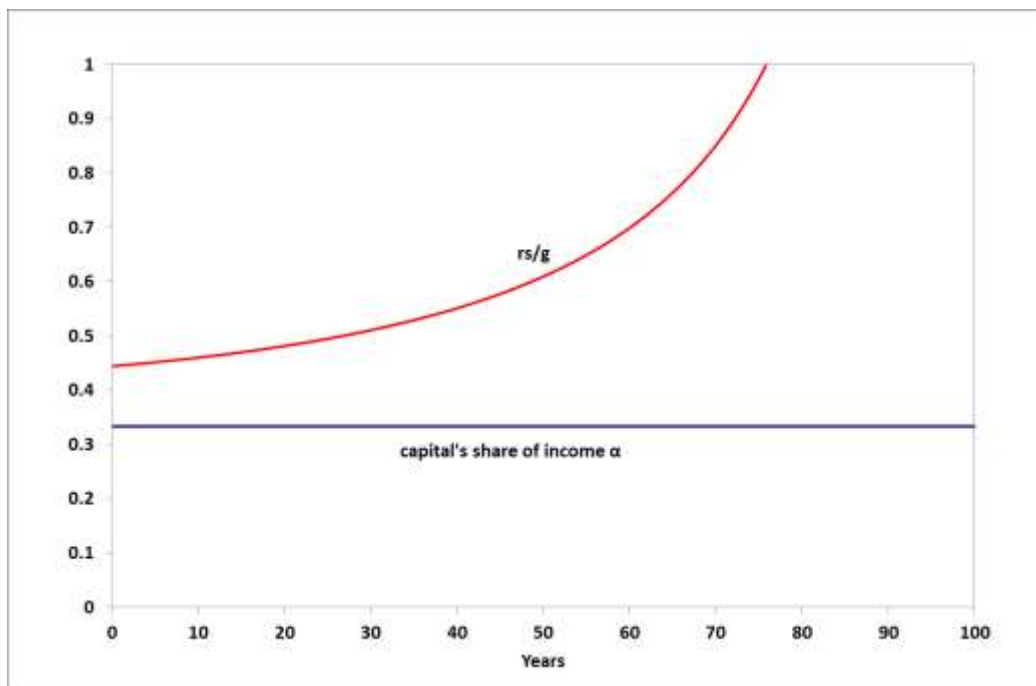


**Figure 2b: Long-term convergence of capital's share of income with  $s$  and  $g$  held constant**

It is worth remarking that the capital to income ratio  $\beta$  clearly converges *towards* the ratio  $s/g$  (Figure 2a). However, Figure 2b seems to suggest that, rather than  $\alpha$  converging towards the ratio  $rs/g$ , the ratio  $rs/g$  converges towards  $\alpha$ . This is because of a particular feature of our initial values, the choice  $\sigma = 1$ . In these circumstances, as we noted above, the rate of return on capital (calculated as the marginal productivity of capital) moves in such a way as to exactly offset the increase in the capital to income ratio and keep capital's share of income constant. Interestingly, this remains the case whatever happens to the growth rate. So for instance, in Figure 3, we allow the growth rate  $g$  to decline to zero. The ratio  $s/g$  therefore goes to infinity over the course of the run. As expected, the capital to income ratio  $\beta$  rises substantially (Figure 3a) more than doubling to reach around 9 by the end of the run. It is comforting to note, however, that it does not explode uncontrollably, in spite of Piketty's second law. Even more striking is that capital's share of income  $\alpha$  once again remains constant (Figure 3b), because the rate of return  $r$  falls exactly fast enough to offset the rise in the capital to income ratio.



**Figure 3a: Long-term behaviour of the capital to income ratio as  $g$  goes to zero ( $\sigma=1$ )**



**Figure 3b: Long-term behaviour of capital's share of income as  $g$  goes to zero ( $\sigma=1$ )**

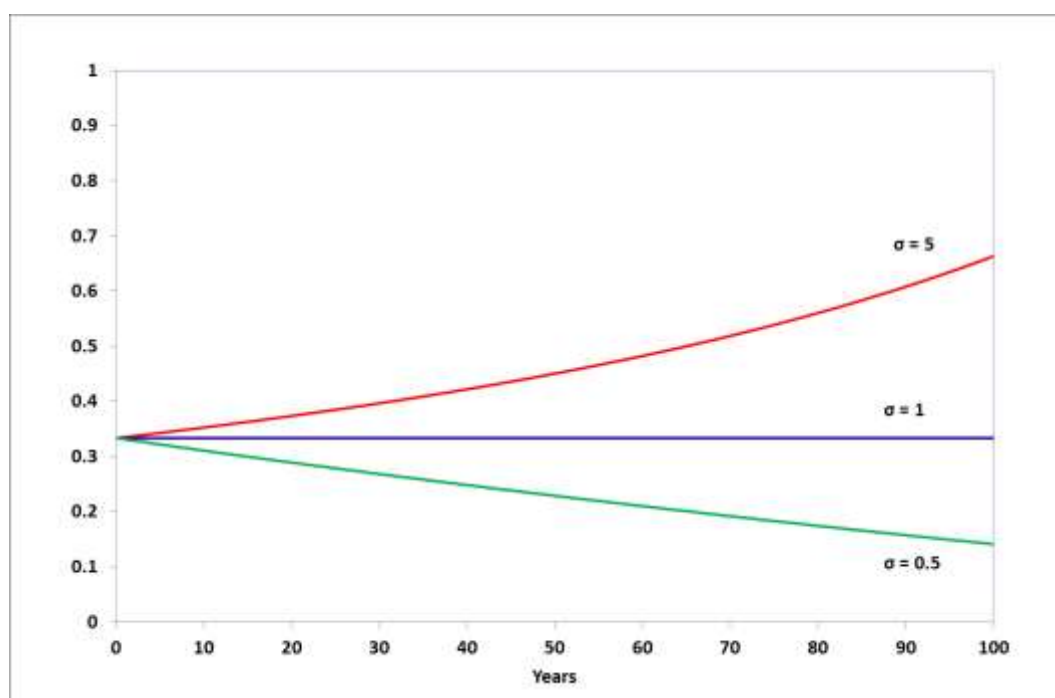
Notice that this lack of convergence of  $\alpha$  towards  $rs/g$  is not a refutation of Piketty's law, since  $g$  is not held constant over the run. This result does go some way, however, to mitigate fears of an explosive increase in inequality as growth rates decline. Indeed, as Figure 3b makes clear, if the elasticity of substitution  $\sigma$  is exactly one, then the decline of the growth rate to zero has no impact at all on capital's share of income.<sup>10</sup>

<sup>10</sup> This result (the constancy of capital's share of income) holds irrespective of the assumed behaviour of the savings rate  $s$ . Note however that there is a wide range of possible variations on the capital to income ratio, when the savings rate is allowed to vary. For instance, if the savings rate goes to zero along with the



The stability of capital's share of income only holds, however, when the elasticity of substitution between labour and capital is equal to one. Figure 4 illustrates the outcome of the same scenario ( $g \rightarrow 0$ ) on capital's share of income for three different values of  $\sigma$ : 0.5, 1 and 5 (see Appendix 1). As predicted, when the elasticity of substitution  $\sigma$  rises above one, capital's share of income increases. Indeed, when  $\sigma$  equals 5, capital's share approaches 70% of the total income. Piketty notes (2014b: 39) that the (less dramatic) increases in capital's share of income visible in the data over the last decades are consistent with an elasticity in the region of 1.3 to 1.6.

Conversely, however, with an elasticity of substitution less than 1, capital's share of income declines over the period of the run, in spite of the fact that both  $s/g$  and  $rs/g$  go to infinity. This is an important finding from the point of view of our aim in this paper. To re-iterate, there is no necessarily inverse relationship between the decline in growth and the share of income to capital. Rather, the impact of declining growth on capital's share of income depends crucially on the rate of return on capital which depends in turn on technological and institutional structure. Specifically, with an elasticity of substitution between labour and capital less than one, and capital remunerated according to its marginal productivity, declining growth can perfectly well be associated with an increase in the share of income going to labour.



**Figure 4: Long-term behaviour of capital's share of income as  $\sigma$  varies ( $g \rightarrow 0$ )**

This theoretical result is not particularly insightful without an adequate account of the relationship between capital's share of income and the distribution of ownership of capital assets. Under the conditions of our reference case, both income and wealth are equally distributed between workers and capitalists. For all of the scenarios so far elucidated, the

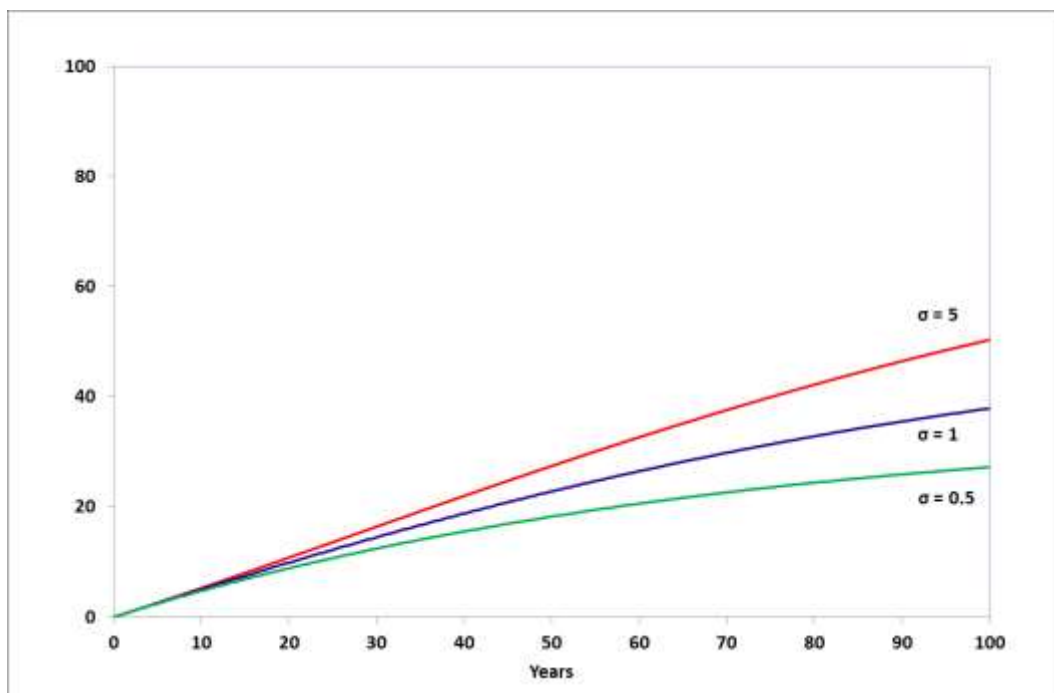
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growth rate, then the ratio  $s/g$  is constant over the run. The capital to income ratio rises very slightly but as before capital's share of income remains constant.

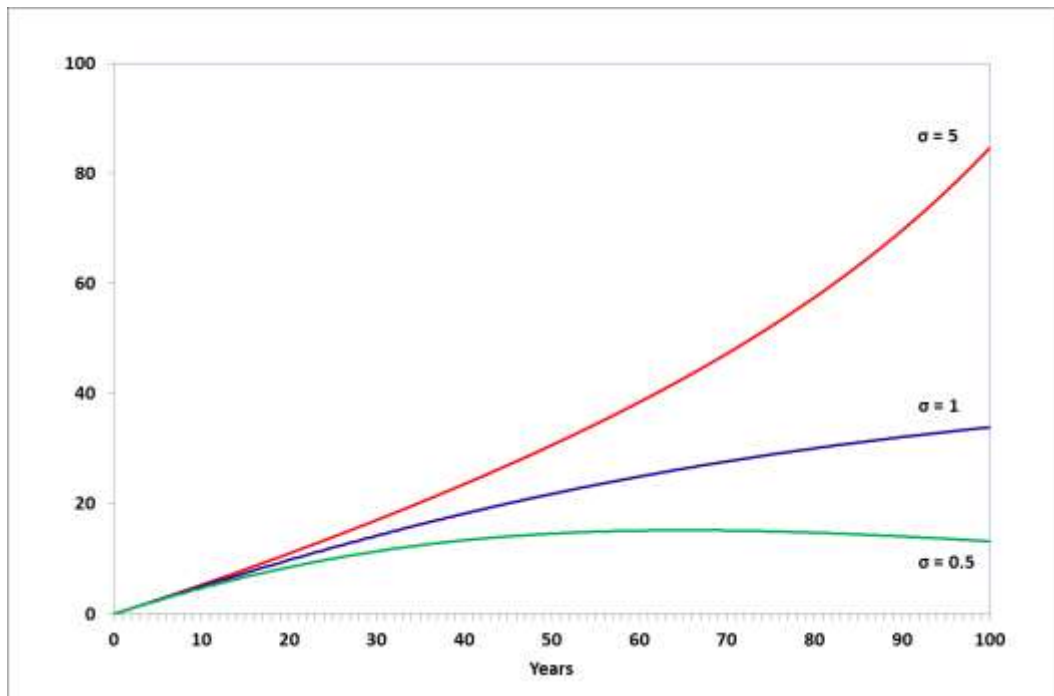
inequality index therefore remains unchanged – and equal to zero. There is no inequality in such a society, whatever happens to the share of income going to capital.

Clearly of course, this is not very realistic as a depiction of capitalist society. One of the things we know for sure, not least from Piketty’s work, is that the distribution of both wealth and wages is already skewed in modern societies, sometimes quite excessively. One element in that dynamic is the savings rate  $\sigma$ . It is well-documented that the propensity to save is higher in high income groups than in low income groups. Kalecki (1939) proposed that the propensity to save amongst workers was zero and for the lowest income groups in the UK, the data support this view (ONS 2014).

For illustrative purposes, we suppose next that – for whatever reason – the savings rate amongst workers is lower than the national average, at 5% of disposable income. The savings rate of capitalists rises (equation 18) to ensure that the overall savings rate across the economy remains at 8%. Figure 5 shows that this apparently trivial innovation has the immediate effect of introducing income inequality, without any decline in the growth rate and with an entirely equal initial distribution of ownership. In Figure 5a, incomes amongst capitalists are up to 50% higher than those amongst workers by the end of the period. This is a fascinating insight into the structural dynamics through which capitalism has an in-built function for the divergence of incomes (Kalecki 1939, Kaldor 1955, Galbraith 2013).



**Figure 5a: Inequality in incomes under differential savings rate ( $g=2\%$ )**

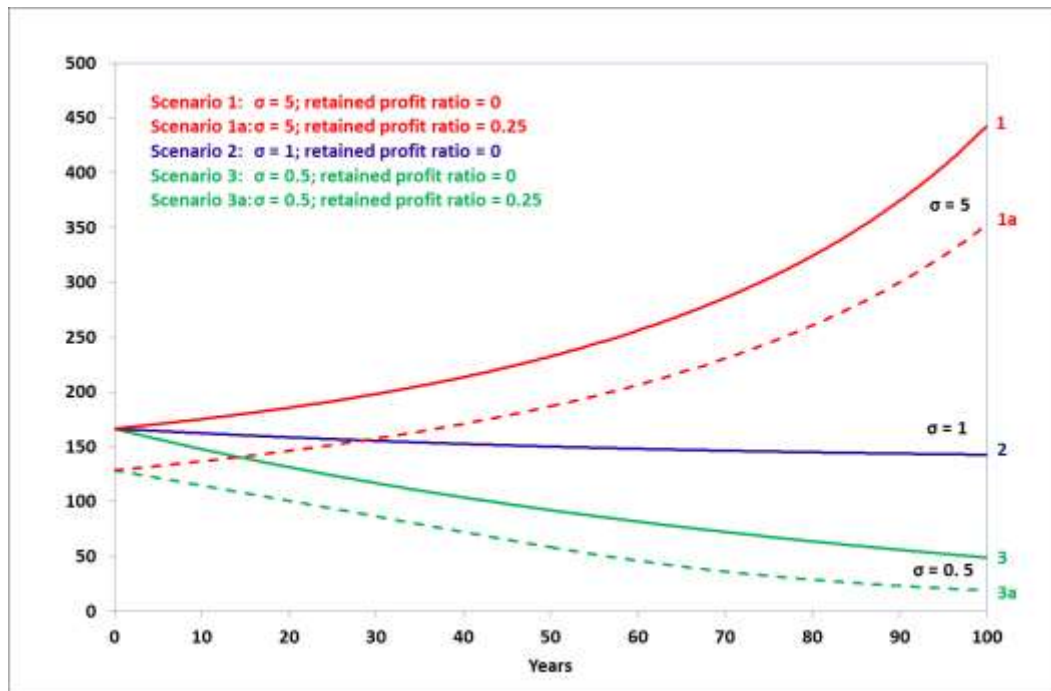


**Figure 5b: Inequality in incomes under differential savings rate ( $g \rightarrow 0$ )**

Under conditions of slowing growth (Figure 5b), an interesting phenomenon emerges. For high  $\sigma$  (ie high substitutability of capital and labour), the inequality between capitalists and workers is exacerbated. When  $\sigma = 5$ , capitalist incomes are over 80% higher than worker incomes by the end of the scenario. By contrast, this situation is significantly ameliorated for low  $\sigma$ . Capitalist incomes are less than 15% above worker incomes at the end of the run when  $\sigma$  is equal to 0.5 and inequality is declining.

The increases in inequality shown in Figures 5a and 5b are stimulated simply by changing the savings rate, assuming a completely equal distribution of income and capital at the outset. Figure 6 illustrates the outcome, once we incorporate inequality in the initial distribution of assets. For the purposes of this illustration, we assume that capitalists comprise only 20% of the population but own 80% of the wealth – a proportion not massively unrealistic from the perspective of today’s global distribution (ONS 2014, Oxfam 2015).

For the scenarios in Figure 6, we also assume (rather conservatively) that the distribution of wages remains equal between the two groups, despite the skewed distribution in asset ownership: capitalists earn 20% of the wages and workers earn 80%. Capitalist incomes are nonetheless immediately around 166% higher than workers because of their additional income from returns to capital. What happens subsequently depends crucially on the value of  $\sigma$ . With high  $\sigma$ , inequality rises steeply as capitalists protect returns to capital by substituting away from expensive labour. So for instance, when  $\sigma$  equals 5 (Scenario 1 in Figure 6), capitalist incomes are almost 450% higher than worker incomes by the end of the run. With low values of  $\sigma$ , however, it is again possible to reverse the initial inequality, bringing the income differential down until, for  $\sigma$  equal to 0.5 (Scenario 3), capitalist incomes are less than 50% higher than worker incomes.



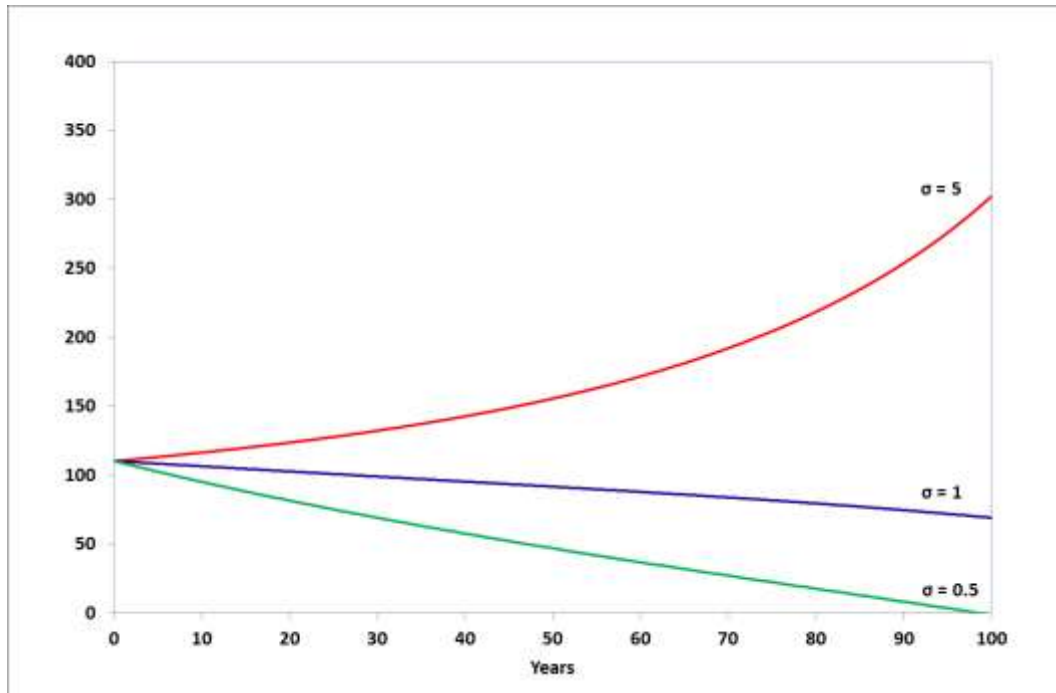
**Figure 6: Income inequality with skewed initial ownership and differential savings**

In all the simulations described so far, the retained profits of firms are assumed to be zero. Figure 6 shows two additional scenarios (1a and 3a), in which this default assumption is relaxed, and firms are deemed to retain 10% of their profits to finance net investment. The impact of this assumption on inequality is significant, particularly for high values of  $\sigma$ , where capitalist incomes are reduced from 450% to around 350% of worker incomes. The impact is lower for low values of  $\sigma$ . Essentially, increasing the retained profits of firms has three related impacts on household finances. Firstly, it reduces the return to capital by lowering the distributed profits from firms. Secondly, it reduces the financing requirement of firms, who consequently issue less new equity and require less debt. Less debt for firms also means fewer deposits for households (equation 26). Taken together with the lower requirement for equity this leads to a lower net worth for households. Given differential savings rate and an unequal distribution of assets, the impact of these changes is greater on capitalist households than on worker households.

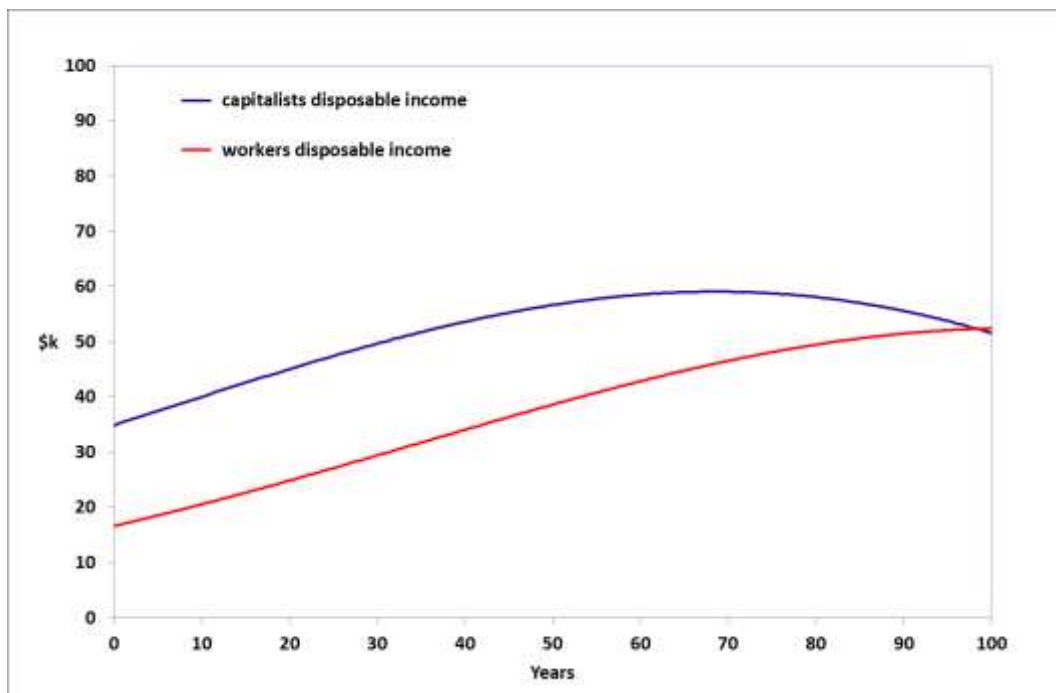
Finally, we explore the possibilities of addressing rising inequality through progressive taxation. It is clear immediately that this task will be much easier when the underlying structural inequality rises less steeply than when it escalates according to the  $\sigma = 5$  scenario in Figure 6. In fact, as Figure 7a illustrates, a modest tax differential (a tax band of 40% applied to earnings higher than the income of workers) and a minimal wealth tax (of only 1.25% in this example) when taken together could equalise incomes relatively easily when  $\sigma = 0.5$  but fails to curb the rising inequality when  $\sigma = 5$ .

Figure 7b shows the per capita disposable incomes of the two segments for the low elasticity case. It is notable that towards the end of the run, capitalist incomes and worker incomes are at more or less the same level even though the overall growth rate has declined to zero, exactly counter to the fear of rampant inequality from declining growth rates which motivated this study. Indeed, extension of the model run beyond 100 years would see workers incomes overtake capitalist incomes under these assumptions. Essentially, workers

and capitalists would have swapped places in distributional terms. Presumably, this would already have triggered corrective policy responses – such as a reduction in the level of wealth tax, or indeed a reduction in the savings rate. There are interesting parallels here to the situation Keynes’ characterised in the last chapter of the *General Theory* as ‘the euthanasia of the rentier’, in which a persistent oversupply of savings leads to a progressive decline in the rate of return on capital (Keynes 1936).



**Figure 7a: Inequality reduction through progressive taxation**



**Figure 7b: Convergence of incomes under progressive tax policy ( $g \rightarrow 0$ ;  $\sigma = 0.5$ )**

## Discussion

In his bestselling book, *Capital in the 21<sup>st</sup> Century*, French economist Thomas Piketty has proposed a simple and potentially worrying thesis. Declining growth rates, he suggests, give rise to worsening inequalities. This thesis is particularly challenging for ecological economists. Motivated by a combination of social and ecological concerns, many ecological economists tend to be critical of growth-based economics. Some have argued forcefully for a profound shift in economic policy away from the pursuit of the GDP as an indicator of progress, and towards a different kind of macroeconomics. Our own prior work exemplifies this argument (Victor 2008, Jackson 2009). For us, the suggestion that declining growth rates precipitate inequality is a challenge that has to be taken extremely seriously.

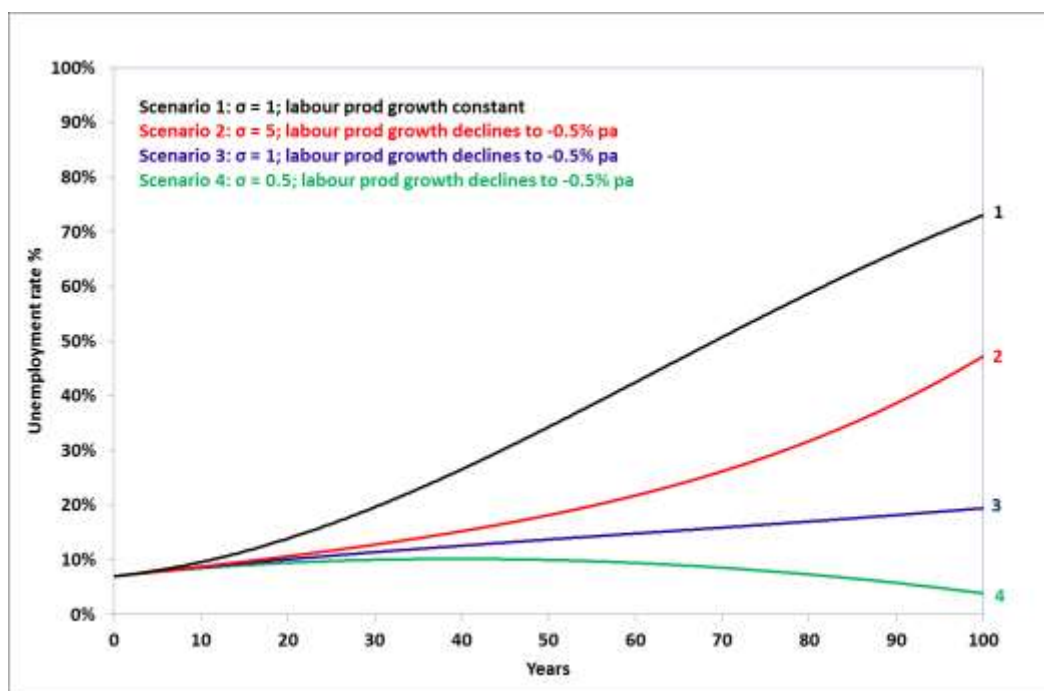
What we have shown in this paper is that under certain conditions it is indeed possible for income inequality to rise as growth rates decline. However, we have also established that there is *absolutely no inevitability at all* that a declining growth rate leads to explosive (or even increasing) levels of inequality. Even under a highly-skewed initial distribution of ownership of productive assets, it is entirely possible to envisage scenarios in which incomes converge over the longer-term, with relatively modest intervention from progressive taxation policies.

The most critical factor in this dynamic is the level of substitutability between labour and capital. Higher levels of substitutability ( $\sigma > 1$ ) do indeed exhibit the kind of rapid increases in inequality predicted by Piketty, as growth rates decline. In an economy with a lower elasticity of substitution ( $0 < \sigma < 1$ ), the dangers are much less acute. More rigid capital-labour divisions appear to reinforce our ability to reduce societal inequality.

From a conventional economic viewpoint, this might appear to be cold comfort. Lower values of  $\sigma$  are often equated with lower levels of development. As Piketty points out (2014a: 222), low levels of elasticity characterised traditional agricultural societies. Other authors have suggested that the direction of modern development, in general, is associated with rising elasticities between labour and capital (Karagiannis et al 2005, eg). The suggestion seems to be that progress comprises more of the same.

It is however an open question whether this is necessarily the case. The contention that progress moves inevitably in the direction of higher  $\sigma$  embodies numerous ideological assumptions. In particular it seems to be consistent with a particular form of capitalism that has characterised the post-war period: a form of capitalism that has come under increasing scrutiny for its potent failures, not the least of which is the extent to which it has presided over continuing inequality.

The possibility of re-examining this assumption resonates strongly with other suggestions for a more sustainable economic model. In our own work, for example, we have highlighted the importance of labour-intensive services both in reducing material burdens across society and also in creating employment in the face of declining growth (Jackson 2009; Jackson and Victor 2011). The challenge of maintaining full employment under declining growth is particularly profound. In fact, under the scenarios developed in the previous section, if labour productivity growth is constant throughout the run then unemployment rises to over 70% (Figure 8: scenario 1), a situation that would clearly be disastrous for any society.



**Figure 8: Unemployment scenarios under declining growth**

Suppose however, that labour productivity did not continually grow at a constant 1.8% per annum. We have argued elsewhere (Jackson 2012, Jackson and Victor 2011) that challenging assumptions about labour productivity constitutes an important avenue of opportunity for structural change in pursuit of sustainability. Instead of a relentless pursuit of ever-increasing labour productivity, economic policy would aim to protect employment as a priority and recognise that the time spent in labour is a vital component of the value of many economic activities. The suggestion here is that there may be employment opportunities to be had through a structural transition to more labour intensive sectors of the economy. This would make particular sense for service-based activities – for instance in the care, craft and cultural sectors – where the value of the activities resides largely in the time people devote to them. It would of course involve protecting the quality and intensity of people’s time in the workplace from the interests of aggressive capital. Such a proposal is not a million miles from Minsky’s (1986) suggestion that government should act as ‘employer of last resort’ in stabilising an unstable economy.

Scenarios 2 to 4 in Figure 8 all describe a situation in which by the end of the run, labour productivity growth has declined to a point where it is very slightly negative. At this point labour productivity is actually falling in the economy. Figure 8 reveals that this decline in labour productivity growth is not in itself sufficient to ensure acceptable levels of unemployment. For higher values of  $\sigma$ , unemployment is still running dangerously high. But for lower values of  $\sigma$  it is possible not only to maintain but even improve the level of employment in the economy, in spite of a decline in the growth rate to zero.

Up to this point, our analysis of the elasticity of substitution has been a broadly descriptive one. We have explored the influence of the elasticity of substitution between labour and capital on the evolution of inequality (and employment) in an economy in which the growth rate declines over time. It would be wrong to conclude from this that we are able to alter

this elasticity at will. Most conventional analyses assume that values of  $\sigma$  are given – an inherent property of a particular economy or state of development. Such analyses usually confine themselves to showing how allowing for a range of elasticity facilitates a better econometric description of a particular economy than assuming an elasticity of 1. Our own analysis here also assumes that the elasticities themselves are fixed features of the economy over time. The production function in equation 21 is predicated precisely on this assumption.

There is however a tantalising suggestion inherent in this analysis that changing the elasticity of substitution between labour and capital offers another potential avenue towards a more sustainable macro-economy, and in particular a way of mitigating the pernicious impacts of inequality and unemployment in a low growth economy. Exploring that suggestion fully is beyond the scope of this paper, but is certainly worth flagging here. It would require us first to move beyond the CES production function formulation adopted here.

The appropriate functional form for such an exercise would be a Variable Elasticity of Substitution (VES) production function. We note here that there is substantial justification and considerable precedent for such a function (Sato and Hoffman 1968, Revankar 1971). Antony (2009) suggests that VES functions offer better descriptions of real economies than either CES or Cobb-Douglas functions. Adopting such a function would allow us to explore scenarios in which  $\sigma$  changes over time. This possibility is the subject of ongoing research.

We should also recall here our assumption that the rate of return to capital is equal to the marginal productivity of capital. As we remarked earlier, this assumption only holds in markets conditions where capital is unable to use its power to command a higher share of income. Clearly, in some of the scenarios we have envisaged, this assumption may no longer hold. Where political power accumulates alongside the accumulation of capital, the danger of rising inequality is particularly severe and is no longer offset simply by changes in the economic structure. This question also warrants further analysis.

In summary, this paper has explored the relationship between growth, savings and income inequality, under a variety of assumptions about the nature and structure of the economy. Our principal finding is that rising inequality is by no means inevitable, even in the context of declining growth rates.



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## Appendix 1: Initial Values for the SIGMA Model

Sources for data: see note 9.

<b>Variable</b>	<b>Values</b>	<b>Units</b>	<b>Remarks</b>
Initial GDP	1,800	\$billion	UK GDP is currently around £1.6 trillion; Canada GDP is around CAN\$1.9 trillion.
Initial national Income	1,500	\$billion	UK and Canadian NI are both around 17% lower than the GDP.
Initial capital stock K	4,500	\$billion	Based on the estimate of capital to income ratio chosen below.
Initial capital to income ratio $\beta$	3		Capital to income ratio in Canada is a little under 3; in UK it is higher at around 5.
Initial capital share of income $\alpha$	33%	%	The wage share of income as a proportion of NI is around 60% in both Canada and the UK, implying that the capital share of income is around 40%. We have chosen a slightly lower value here to be compatible with the full range of values of $\sigma$ chosen in different scenarios.
Initial savings rate $s$ as percentage of National Income	8%	%	The ratio of net private investment to national income in Canada was around 8% in 2012. In the UK the number was somewhat lower.
Elasticity of substitution $\sigma$ between labour and capital	varies 0.5 - 5		In theory $\sigma$ can vary between 0 and infinity. Empirical values found in the literature typically range from 0.5 (Chirinko 2008) up to around 10 (Pereira 2003, Duffy and Papageorgiou 2000). A lower value of 0.5 and upper value of 5 is sufficient to demonstrate divergent conditions here.
Population	50	Million	The population of Canada is 34 million; that of the UK just over 60 million.
Workforce as % of population	50%	%	Workforces in developed nations are typically between 45% and 55% of the population.
Initial workers as % of population	50%	%	Initially there is no distinction between 'workers' and 'capitalists'.
Initial % of wages going to workers	50%	%	Initially there is no distinction between 'workers' and 'capitalists'.
Initial % of capital owned by capitalists	50%	%	Initially there is no distinction between 'workers' and 'capitalists'.
Initial unemployment rate	7%	%	Typical of both Canada and the UK over the last few years.
Distribution parameter $a$	varies		This value is calibrated for each $\sigma$ according to equation (17) at time $t = 0$ .
Initial technology augmentation coefficient $A_0$	varies		This value is calibrated for each $\sigma$ (and $a$ ) using the production function at time $t = 0$ .
Initial growth rate $g$ in reference scenario	2%	%	Growth rates (of GDP) in both the UK and Canada were slower than this in the aftermath of the financial crisis and in the UK currently a little higher.
Initial growth in labour productivity in reference scenario	1.8%	%	This value is consistent with a 2% rate of growth in the NI and the maintenance of a constant employment rate when $\sigma = 1$ .
Initial tax rates	25%	%	In the reference scenario, typical economy wide net taxation rates (as a percentage of household disposable income) are applied to the incomes of both capitalists and workers.
Retained profits ratio	0-25%	%	Default assumption is that retained profits are zero and firms contribution to investment costs is equal only to the depreciation on capital.

